Experimental and Numerical Verification of the Cause of Hopf Bifurcation in a Microwave Doubler

Saswata Basu, Stephen A. Maas, *Fellow, IEEE*, and Tatsuo Itoh, *Fellow, IEEE*

***Abstract*—The appearance of Hopf bifurcation in a microwave doubler is due to the relative speed of the recombination delay of the junction diode with the input RF signal, which results in a dynamical negative resistance. This is verified here experimentally by using a back-to-back diode structure that quenches Hopf bifurcation numerically through stability analysis in the frequency domain.**

I. INTRODUCTION

FREQUENCY multipliers in junction varactor and step-recovery diodes are well known for their spurious-oscillation instabilities [1], [2]. No work has been done in characterizing and predicting this type of instability. In addition, there has been no report of a breakdown of the "spurious oscillation" to chaos. This paper attempts to provide experimental and numerical verification of chaos observed in a 5–10 GHz frequency doubler. With the input power level fixed, the bias level of the junction diode was increased gradually. At a critical bias level, a mixed product spectrum was observed denoting the presence of two frequencies, input and self-generated. The appearance of a spurious oscillation due to a varying parameter is known as the Hopf bifurcation phenomenon; the parameter itself is referred to as the "bifurcation" parameter. The magnitude of this oscillation is low and increases gradually with the bifurcation parameter; this is a "soft" Hopf bifurcation [3]. Further increase results suddenly in a secondary Hopf bifurcation, and the system enters a three-frequency quasiperiodic regime. The magnitude of all the mixed products suddenly increases; this is a "hard" Hopf bifurcation [3]. A further increase results in a chaotic breakdown manifested by a broadband noise around the spectral lines. These bifurcations occur whenever the combination of the input power level and the bias level exceeds a certain instantaneous limit. The reason for this will become apparent below. Here's a look at the bifurcation sequence: With a fixed input power level of 16 dBm, primary bifurcation occurs at a bias level of -2.59 V, followed by secondary bifurcation at -1.33 V, and chaos at -0.33 V. The above path to chaos is classically known as the quasiperiodic route, where the system undergoes two Hopf bifurcations, after which chaos becomes likely to occur [4].

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S. Basu is with Cascade Microtech, Inc., Beaverton, OR 97005 USA.
T. Itoh is with the University of California, Los Angeles, CA 90024 USA.
S. A. Maas is with Nonlinear Technologies, Inc., Long Beach, CA 90807 USA.
IEEE Log Number 9413480.

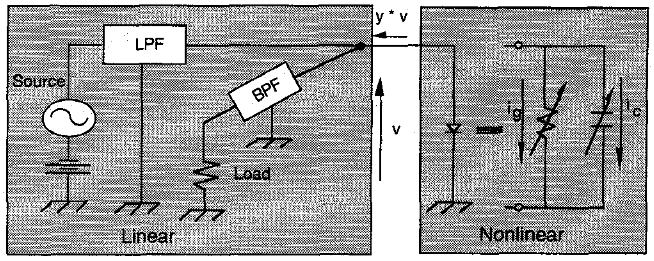

Fig. 1. A harmonic balance analysis of the frequency doubler.

The observed Hopf bifurcation is primarily due to the finite recombination lifetime of the minority carriers of the pn junction diode. When the forward-biased pn junction diode is suddenly reverse biased, the current continues to flow in the forward direction because of the recombination time delay of the minority carriers. Consequently, a dynamical negative resistance is set up whenever the diode is forward biased. Since the forward-bias voltage is the sum of the bias and the input source voltage, the two bifurcation parameters essentially behave as one. Hence, through either parametric variation chaos can be reached.

The negative resistance effect of a junction diode has been studied earlier in the 1960's under the topic of parametric amplification [5]. However, this parametric effect is different from the work discussed here in the sense that a pump frequency and a small-amplitude signal frequency are simultaneously applied to make the device behave like a time-varying linear capacitance at the signal frequency. When the current is permitted to exist at the idler frequency $\omega_p - \omega_s$, further mixing occurs and results in a power transfer from the pump signal to the RF signal, which is interpreted as an equivalent negative resistance.

II. FORMULATION

The analysis of the doubler is based on the piecewise harmonic balance method [6]. The stability analysis follows the procedure in [7], but the formulation given here is believed simpler and still applicable to practical microwave circuits. Without loss of generality, a representative nonlinear device can be modeled as a parallel combination of nonlinear resistor and a nonlinear capacitor; this model happens to be adequate for characterizing diodes. The Y-parameters of the linear portion of the circuit is obtained using the microwave simulator MDS [8]. The harmonic balance equation at the junction of

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the linear/nonlinear network in Fig. 1 is given by Kirchoff's circuital law

$$E(V_k) = I_{g,k} + I_{c,k} + Y(k\omega_o) \cdot V_k \qquad (1)$$

where E is the error function that needs to be reduced to zero through some iterative mechanism.

The diagram in Fig. 1 depicts the frequency doubler network separated piecewise into linear and nonlinear network. The structure is that of a typical doubler, with a low-pass filter (LPF) at 5 GHz input signal and a band-pass filter (BPF) at 10 GHz, each filter presenting a high impedance at the other frequency looking in from the diode. The tuning elements and bias circuit is not shown in the diagram for clarity.

In order to extract local stability behavior of the system, we need to perturb the solution V_k in amplitude and frequency. The perturbed equation is in the form

$$\sum_k \sum_1 J_{S,k-1} \cdot \Delta V_k = 0 \qquad (2)$$

where

$$J_{S,k-1} = [G_{k-1} + (s + \Omega_{k,1})C_{k-1} + Y(\Omega_{k,1} - js)] \qquad (3)$$

is defined as the Stability Jacobian, J_S, which becomes the ordinary Jacobian of (1) for $s = 0$ (unperturbed state) and is used in the Newton-Raphson method for solving (1). This Jacobian consists of the nonlinear conductance matrix, the nonlinear capacitance matrix, and the admittance matrix of the linear circuit. Following the Nyquist approach in [7], the determinant of (4) is modified by a factor of $\exp(-\pi s/\omega_o)$ to remove the singularity at infinity and plotted only in the frequency range $[0, \omega_o/2]$ to extract the stability information. The Nyquist criterion for instability is

$$N_o = Z; \quad \Rightarrow \text{stable if and only if } N_o = 0 \qquad (4)$$

where N_o is the total number of encirclements of the origin and Z is the number of unstable zeros on the right-hand plane (RHP) of the s-plane.

The bifurcation parameter, for example the input power or the bias level, is stepped gradually up from a low level. At each solution point determined by the harmonic balance analysis, the local stability at that point is found by using the Nyquist approach described above. In this way, a large signal stability analysis is carried out.

In the following sections, experimental and numerical verification of the onset of Hopf bifurcation in the doubler is described.

III. Experimental Verification

The chaotic doubler circuit was slightly modified by incorporating an additional diode in a back-to-back series connected fashion, as depicted in Fig. 2. The choice of the anode or cathode of the diodes facing outward in the back-to-back structure is not relevant to the experiment and will give similar results. In this structure one of the diodes stay reverse biased all the time. Because of the series connection, the structure is never in the forward-biased mode. Consequently, there is no recombination time delay effect, and hence spurious oscillations are not observed.

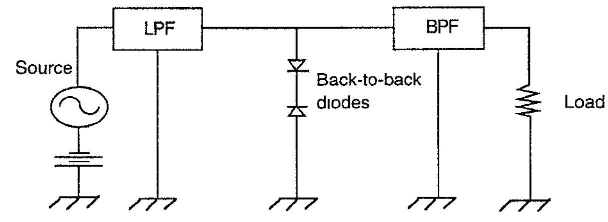

Fig. 2. A back-to-back two-diode nonbifurcating frequency doubler circuit.

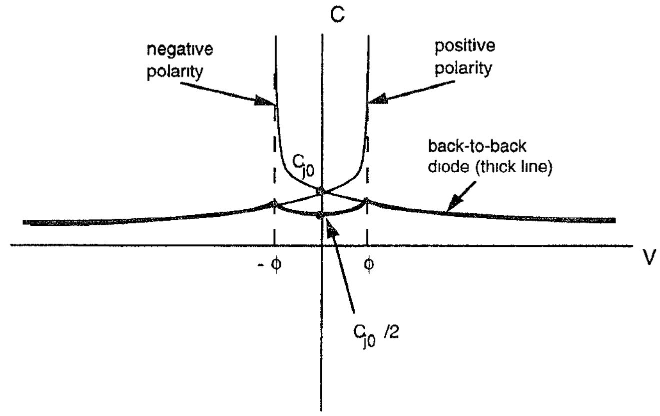

Fig. 3. The C–V curves of a positive polarity diode, negative polarity diode, and a series combination of two back-to-back diodes.

The back-to-back doubler still provides a conversion, but not as good as a single-diode doubler. The conversion results from the nonlinearity of the charge or capacitance of the two diodes. The C–V curves of the two diodes, one forward biased, the other reverse biased, and their combined effect are shown in Fig. 3. At the built-in potential, the capacitance of the forward-biased diode approaches infinity due to the closure of the depletion region. The combined series capacitance becomes essentially that of the reverse-biased diode. At 0 V, the total capacitance is half that of a single diode. As a result, the variation of capacitance over a wide range of input voltage is much smaller, which makes the frequency conversion weaker. Therefore, even though there are no Hopf bifurcations, the back-to-back structure may not be an optimum choice for frequency multiplication. The conversion can be improved by biasing away from zero volts because of the C–V asymmetry presented to the input signal.

IV. Numerical Verification

The primary Hopf bifurcation can be predicted based on a single-tone stability analysis as described in Section II. The experimental value is off by about 1–1.5 dB in terms of the input RF power; this is adequate accuracy considering that quasistatic models were used to model the linear part of the circuit [8]. To determine if the cause of Hopf bifurcation is due to the recombination time delay, the diffusion capacitance was modeled explicitly in terms of the minority carrier lifetime, τ, of the diode as

$$C = \tau \cdot \frac{dI(v)}{dt} \qquad (5)$$

where the current $I(v)$ is given by the familiar Shockley diode equation. According to the manufacturer specifications of the diode (Alpha DVA6735), τ is about 10 ns. With this value the predicted bifurcation point is about 1–1.5 dB off from the experimental result, where the bifurcation parameter used is the input power level. This discrepancy is acceptable since quasistatic models used in simulating the linear part of the circuit have limited accuracy bandwidth. When this delay is made negligibly small, for example $\tau = 6$ ps, no numerical Hopf bifurcation appears. This is because very little charge is stored in 3/100th period of the input signal, $T(f = 5$ GHz). When the delay is made bigger to about 3/10 T, then Hopf bifurcation registers, although at a higher bifurcation parameter value. As the delay is made closer to its norm of 10 ns the occurrence of Hopf bifurcations registers closer to the experimentally observed value. No change in the Hopf point was observed if the delay was changed by a factor of 10 about its norm; hence, the predicted Hopf point is quite stable within manufacturer tolerances of the delay. Thus, the dependence of Hopf bifurcation on the minority carrier lifetime of the junction diode is clear from the above numerical results. Furthermore, this single-tone stability analysis can still be used for further increment of input power or bias level to detect a secondary Hopf bifurcation, even after the initial one. This is valid only in this case, since it is *a priori* known experimentally that the spurious tones were about 40 dB below the 10-GHz spectral output and, therefore, can be neglected; hence, a multitone analysis is not required. Numerically and experimentally, a secondary Hopf bifurcation occurs within 1.2-dB increase in the input power level (bifurcation parameter) showing that the system does bifurcate into a three-frequency quasiperiodic regime. In [4], it has been theoretically and experimentally proven that when a system enters a three-frequency quasiperiodic regime it has the potential to disintegrate to chaos. In chaos literature, it has been numerically shown to occur for three-order autonomous circuits [9], but to prove its occurrence in complex circuits such as the frequency doubler is difficult and has not yet been done. Hence, the best we have numerically achieved is to determine when the system goes into a three-frequency regime.

V. Conclusion

In this paper, we have verified that the onset of Hopf bifurcation is primarily due to the dynamical negative resistance manifested by the minority carrier lifetime of the pn junction diode. The quasiperiodic path to chaos has also been numerically verified for this doubler through a single tone stability analysis.

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